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Magical Numbers May Govern the Optimum Size of Curriculum Classes

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Abstract. *Frequently, the public education institutions are yearly financed, based upon the total number of students, enrolled by those institutions, at the beginning of the academic year. The social demand to increase the number of graduates prompts enlarging the size of classes, within the upper limits imposed by the infrastructure, by pedagogic, ergonomic or administrative regulations, size, usually, less connected with the results of an entire cycle. But, how to choose the size of a class, to ensure both the maximum financial support and the minimum number of graduates expected by the society ? The authors suggest that a criterion for finding the optimum size of classes be connected with the their academic results through the curriculum' cycle, f. e. with the ratio graduated seniors / enrolled freshmen, result to comply with accreditation conditions for passing between years. Thus, there may be numerically determined the optimum size of a class of freshmen, governed by "magical numbers", specific to the legal accreditation rules in force.*

Keywords: Accreditation rules, *econophysics*, education funding. Magical numbers, optimum size of curriculum classes.

1. Introduction

The management of education, at different levels, rises, frequently, questions about how to decrease the cost of education, per student. This aspect is particularly important when the expenses of an education institution (E.I.), university, school, are integrally supported by the public budget, national or local and when the society needs more graduates.

In many countries, including Romania, the public education institutions are yearly financed based upon the total number of students (respectively, pupils) enrolled with those institutions, at the beginning of the academic (school) year.

The yearly expenditures of an E.I. consist of fixed and of variable ones.

The variable costs for the E.I. are those proportional with the number of students.

The fixed costs which, by name, would be independent of the number of students, are, finally, for a rather large education unit, also roughly proportional, with the total number of students enrolled in that institution, subject to a small relative error (of the same order of magnitude with the relative error on the variable costs), for an institution having ~ 50 classes, the relative error of appearing or disappearing a class is ~ 2%, value acceptable for the relative error on variable expenses.

Each year, the E. I. tries to enroll the maximum, possible for it, cohort of students and tries to divide all enrolled students in classes which, usually, to have the standard allowed maximum size. The fixed expenditures of an E.I. are connected with the number and the size of classes. Larger the class, smaller their number less fixed costs per student, but there are limits of the upper size of a class, imposed by the E.I. infrastructure, by pedagogic, ergonomic or administrative regulations.

Usually, there are provisions in the rules of Ministry of Education or of other regulatory bodies, about the size of a class, depending of the type of activity: f. e. 80-120 attendants for a lecture, 20-30 students for a tutorial, 10-15 students for a laboratory work, 5- 10 students in a plastic arts class.

How to choose the size of a class, to ensure, both, the highest financial support and at least the minimum number of graduates expected by the society from a given curriculum?

There is needed a criterion for optimum.

2. A criterion for the optimum size of classes

The authors suggest that this criterion for the optimum size of classes of a curriculum be connected with the result of education (training) through that curriculum, for example with the

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ratio graduated seniors / enrolled freshmen, for a complete cycle of studies of a given program (curriculum), result to comply with accreditation criteria.

Usually, ministries of education or, eventually, parliaments, set conditions [1] that a curriculum be periodically (T= ~ 5y) accredited and/or financed (yearly) including: conditions for ratio of students passing in the next year, p_a, and a condition for passing the final (graduation) examination, p_g. Not fulfilling these conditions means non accreditation and non financing of that curriculum in the future [1].

Therefore, there exists a supplementary restriction to be dealt with by managers of an education institution, restriction depending of final result and not only of initial conditions.

The authors show that, by applying this criterion of observing accreditation conditions (the conditions for passing academic years and final examination) one can numerically determine the optimum size of a class when enrolling freshmen, to, both, ensure the highest amount of funding and its best use and the fulfillment of the society minimum requirements.

3. Magical numbers

Because the requirements for accreditation are expressed in the provisions of the existing rules as minimum acceptable percentage [1] and because the number of students implied must always be an integer (the upper rounded up integer, resulting from any computations, because the conditions are minimum ones), these conditions, especially for small cohorts, impose thorough choice of the size of a class, optimum size appearing as being limited by "magical numbers" determined by concrete provisions of the accreditation rules.

The determination of the size of the class subjected to accreditation requirements becomes a problem in Statistical Physics or in *Econophysics*.

4. Hypothetical case study

Here following, there is described an example [2] of such sizing of a program of study (curriculum)[2].

4. 1. The Statement of the proposed problem:

"At the Admission competition into a First Cycle ("License") of an Engineering Curriculum (4 years of study + license examination), at a newly created Faculty (let say "AS"), there are, initially, offered, by the University, for the first cohort of the curriculum, 100 places, funded by the State, from the quota given to the whole University. Because of the existing demand and of the results at the admission examination (3 hour tests in Mathematics and in Physics), the Faculty AS asks, from the University, 5 more State funded places, (to reach D_0 = 105 freshmen) taking the 5 public budgeted place from an other faculty, that other faculty not having covered with demands its initial quota.

The University approved that increase.

Soon, the candidate ranking the 106th on AS list, which has an average mark a little smaller than the mark of the 105th candidate, but larger than the minimum average mark for admission in the University, asked to be admitted in the Faculty AS, too. But he was not accepted by the Faculty. This candidate demanded, directly, to the University, to offer Faculty AS one more supplementary place for a newly enrolled student

The University, because the request was not implying for it supplementary public financing, accepted the demand of the 106th candidate, subject to the Faculty AS' decision. But the Faculty AS did not accept the generous offer of the University (to have M_0 = 106 freshmen). The Faculty AS, refused the place additionally funded (place increasing with 1/105 the initial financing of the curriculum, for its freshmen), mentioning the future accreditation conditions.

Explain the two managerial positions, of the Faculty and of the University, considering the funding per year, per enrolled student, **E**, as being constant, during the whole cycle of License studies and observing the conditions for accreditation.

Legal information [1]: Compulsory Conditions for Accreditation of a Curriculum of Studies by ARACIS (the Romanian State

[1] If observed by state authorities, but this may, sometimes, not happen..

[2] The authors proposed this problem, to the participants at the ceremony of inauguration of the headquarters ("DECANAT") of the new Faculty of Applied Sciences of the University "POLITEHNICA" in Bucharest, on 8th of March 2007. This Faculty has been created, in the summer of 2005, as an independent faculty, on the opportunity of implementing Bologna Reform of Higher Education in Romania. The number of candidates enrolled as freshmen in the academic year 2005-6, with the curriculum offered by the new faculty of Applied Sciences was 105 = D_0. The students passing in the second year was 68, corresponding to a $p_1 = 65\%$.

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Accreditation Agency for Higher Education), Annex. I. 3. 3., the Provisions:
-- CNO IV 5: "between two successive years of study, the minimum percentage of passed students to be achieved must be $p_a = 40\%$" (1)
-- CNO IV 10: "the minimum percentage of the students successful in taken the graduation examination must be $p_g = 51\%$". (2)

4.2. Solution of the problem

The strategy of solving the problem is to determine, when observing (1) and (2), the minimum necessary numbers of **graduates** of the curriculum, with License's degree, D_f (for the D_0 desired by the Faculty AS) and respectively, M_f (corresponding to the M_O freshmen, figure not accepted by the Faculty AS) and to compare them.

The accepted results for D_f and M_f are to be the upper next integers of the exactly found values. (3)

There are then to be compared the costs involved versus the changes in the number of enrolled freshmen.

If the number of students would not be integers, the minimum percentage of graduated students, out from the freshmen, compulsorily resulting from accreditation conditions, P_f, would be:

$$P_f = p_1 * p_2 * p_3 * p_4 * p_g = 0.40^4 * 0.51 = = 0.013056, \quad (4)$$

roughly, 1 graduate in about 76-77 freshmen.

The number of graduates, G_D and G_M, starting from the two initial situations, D_0 and respectively, M_0, rounded only at the end of computations, would be:

$$G_D = P_f*D_0 = 0.013056 * 105 = 1.37088 \rightarrow 2 \quad (5)$$

$$G_M = P_f*M_0 = 0.013056 * 106 = 1.383936 \rightarrow 2 \quad (6)$$

That means that, the conditioned final result, expressed in rounded up integers, would be the same for both number D_0 and M_0 of enrolled freshmen: 1 graduate (lower value) or 2 graduates (upper value), depending of the convention of final rounding up (here - 2 graduates for the normal, upper, rounding up).

Based on this model, the Faculty might had accepted the offer of the University, offer increasing its funding for freshmen with 1/105 of initial value, without supplementary obligations at the end of the 4 y + graduation examination cycle.

But, if there is a rounding up to the upper integer, for **each** year of study, the things change significantly!

4. 2. 1. Summary of the data known from the Statement when rounding up to the upper integer

Known data (*input*) :

$D_0 = 105$; $M_0=106$; $p_i = p_a=0.40$; $p_g=0.51$, (7)
$D_i, M_i, D_f, M_f \in N$, (3)
where D_i, M_i are the numbers of students finishing the i^{th} year of study,
i {1, 2, 3, 4} (8)

Data to be found (*output*):

$D_f = ?$; $M_f = ?$ (9)
$T_D = \Sigma D_i = ?$; $T_M = \Sigma M_i = ?$, (10)
where, T is the total number of funded year*student, for the whole cycle.

4. 2. 2. The symbolic solution

$D_{i+1} \geq p_a * D_i$; $M_{i+1} \geq p_a * M_i$ (11)
$D_f \geq p_g * D_4$; $M_f \geq p_g * M_4$ (12)
$D_i, M_i, D_f, M_f \in N$, (3)

4. 2. 3. The numerical Solution

The minimum number of students to pass in the next year (**rounded up to the upper integer**), for 4 years of study are, successively:

D_i: $D_1 = 105*0.40=$ **42**; $D_2 = 16.8 \rightarrow$ **17**; $D_3 = 6.8 \rightarrow$ **7**; $D_4 = 2.8 \rightarrow$ **3** (13)
M_i: $M_1 = 106*0.40= 42.4 \rightarrow$ **43**; $M_2 = 17.2 \rightarrow$ **18**; $M_3 = 7.2 \rightarrow$ **8**; $M_4= 3.2 \rightarrow$ **4** (14)

The minimum number of students to pass the graduate examination and receive diplomas (rounded up to the upper integer):

$D_f \geq p_d * D_4 = 0.51* 3 = 1.53 \rightarrow \mathbf{D_f} =$ = **2** graduates (15)
$M_f \geq p_d * M_4 = 0.51* 4 = 2.02 \rightarrow \mathbf{M_f} =$ = **3** graduates (16)

4. 3. The interpretation of the results

The relative growth of the number of graduates due to curriculum AS, expected by the investor (the University), $\mathbf{r_f}$ is:

$\mathbf{r_f} = (M_f / D_f) - 1 = 3/2 - 1 =$
= **50 %** , (17)

for a relative increased investment in the freshmen, $\mathbf{r_0}$ of:

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$\mathbf{r_0} = (M_0 \ / \ D_0) - 1 = 106 / 105 - 1 =$
$= \mathbf{0{,}95 \ \%}.$ (18)

For the University (the investor) the resulting leverage, $\mathbf{L_0}$, would be:

$\mathbf{L_0 = r_f / r_o = 50\% / 0.95\ \% = 52.63}$
$\rightarrow \sim\mathbf{53}$ **times !** (19)

The cumulated (consolidated) expenditures, during the whole cycle, due to the enrollment of one supplementary freshman, would increase from

$T_D = \Sigma D_i = 174$ year*student*E (20)

to

$T_M = \Sigma M_i = 179$ y*s* E, (21)

relatively, $\mathbf{r_t}$, with **2.73% .** (22)

The Leverage on the total expenditures for getting one more graduate, L_f, would be:

$\mathbf{L_f = r_f \ / \ r_t} = 0{,}5: (5/174) =$ **17.4 times.** (23)

Therefore, the University is highly interested to have enrolled, by the faculty AS, the 106th student.

The Faculty may had looked at the offer of the University as an obligation to got three graduates instead of two (50% more efforts) for that cohort, for an increase in funding of only 0.95% for freshmen and of 2.73%, for the entire cycle. Therefore the Faculty is not wishing to enroll the 106th student, with a view to the coming accreditation inspection, at the end of the cycle.

It was not worth for the Faculty to accept the additional offer of the University.

Because the examinations are internal procedures, the faculty could, eventually, reduce the level of standards for assessments, applied if accepting the 106th freshmen, but the Faculty AS did not want to do that, the Faculty considering the international standards of assessment as being more important. [3]

5. Magical numbers

By further exploring the model along a wider spectrum of enrolled freshmen in a curriculum, for the same accreditation conditions [1], the authors have found that there are some intervals between which the number of necessary graduates for accreditation is the same; specifically to the mentioned accreditation conditions [1], for the intervals:

$D_0 \in$ 1 - 12 $\rightarrow D_f =$ 1 graduate (24)
$D_0 \in$ 13 - 105 $\rightarrow D_f =$ 2 graduates
$D_0 \in$ 106 - 187 $\rightarrow D_f =$ 3 graduates
$D_0 \in$ 188 - 262 $\rightarrow D_f =$ 4 graduates
$D_0 \in$ 263 - 342 $\rightarrow D_f =$ 5 graduates
$D_0 \in$ 343 - 417 $\rightarrow D_f =$ 6 graduates
$D_0 \in$ 418 - 500 $\rightarrow D_f =$ 7 graduates
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Therefore, one may define discrete, stiff transitions in the number of graduates over the magical numbers of freshmen:

12; 105; 187; 262; 342; 417; 500 . . . (25)

From this finding, an **advice for managers** of curricula: observe the upper limits of classes for the freshmen: **do not exceed Magical numbers**!

We have to note that the difference between successive intervals slightly oscillates, but the trend is towards the value $1/P_f = p_1 * p_2 * p_3 * p_4 * p_g = 0.0256*0.51 = 1/0.013056 = \sim 76$ students.

For other accreditation conditions, there are to easily be found other sets of magical numbers.

References:

[1] Ministry of Education, Research and Youth, www.edu.ro/index/php/articles/6746

[2] "METODOLOGIA de evaluare externa, standardele, standardele de referinta si lista indicatorilor de performanta ale Agentiei Române de Asigurare a Calitatii în Invatamântul Superior Bucuresti",
Art. IV. 4.2.4.e: $p_a = 0.40$ (1) and
Art. IV. 4.2.4.j: $p_g = 0.51$ (2)

[1.2.2] "Cerintele Normative Obligatorii in vederea Acreditarii, ale ARACIS – Fisa Vizitei, Anexa Nr. I. 3. 3. a " :
- C. N. O. IV 5 : $p_a = 0.40$ (1) ;
- C. N. O. IV 10 : $p_g = 0.51$ (2).

[3] Radu CHISLEAG - "A Problem of Accreditation", Workshop EDEN I – "Econophysics, a new Exploratory Domain"; University of Pitesti, Romania, March 20, 2008

3 Sometimes, some education institutions diminish the level necessary for passing, especially high schools. Consequently, the Government felt obliged to introduce, a couple of years ago, the final high school examination ("Baccalaureate") as an external examination, based upon national unique tests and external commissions. The first result was that some high schools in Romania, especially from rural area, obtained a null rate of graduation. The second result was large attempts of cheating.

Unfortunately, in Romania, some public high schools with null graduates have not been closed, in spite of the (theoretically) compulsory accreditation provisions.

On such a rural area, the leader of the largest opposition party in Romania has declare to run for a senator' mandate, at next, autumn 2008, parliament elections.