

Permanently Expanding the Use of Standard Laboratory Equipment to New Applications

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Abstract. *One important resource to cheaply improve Experimental Physics training, in schools and universities and to offer more experimental opportunities to the interested pupils and students, is to use the existing standard teaching laboratory equipment to perform new experiments outside the initial scope the equipment has been designed, produced, advertised and sold for. These new experiments may refer to: larger ranges, a better precision, smaller quantities, more complex investigated objects, new phenomena (eventually interdisciplinary) and higher levels of the processing of experimentally got data.*

In the paper, there is described a new application of the standard equipment designed for the measurement, in a Physics didactic laboratory, of the velocity of light in air, based on the modulation of light in intensity. The author has used the existing equipment to determine the two velocities of propagation of light in the core and in the cladding of the optical fibers of a light cable and the corresponding refractive indices. The experiments performed by students, on four types of light cables, show that the determination of velocities of light in light cables is possible with simple equipment, with a small relative error. The processing of experimental data is also improved as compared with the producing company recommendations.

Keywords Light speeds in optical fibers, new uses of laboratory standard equipment, improving precision of measurements, accurate self evaluation of results.

1. Introduction

For decennials, the General Physics Laboratories of the University "POLITEHNICA" in Bucharest have not received financing to buy

new equipment, the old one applying little to electronics and informatics.

This restriction has lead to develop a creative activity of extending the uses of the existing equipment, beyond the initial scope.

This attitude is preserved being applied now, when, with the financial assistance of European Union (where Romania was accepted as a member), during the summer of 2007, has been bought new equipment for the General Physics teaching laboratory.

A new setup, offered for the determination of velocity in air during a standard laboratory work, has been offered to sophomores of UPB, by the laboratory of General Physics.

A possibility to determine the velocity of light would be to measure: the wavelength in the medium (usually around ~ 0.5 micrometers - possibly to be done with the existing equipment) and the frequency of the light wave, $\nu_0 = \sim 6 \cdot 10^{14}$ Hz, this being a too large frequency to be measured in a school or university General Physics laboratory.

Another possibility to determine the velocity of light, would be to measure the time of propagation, t , of a known signal over a measurable distance, l . Because the velocity of light is very high ($2.997 \cdot 10^8$ m/s, in air) and the usual operational distance in a teaching laboratory is in the meter range, distance which may be measured with a standard lineal, with an absolute error of the order of a mm, there is necessary to be measured a time of ~ 3 ns, too small a time for the precision of the equipment available in Physics didactic laboratories.

Other different methods could be used to measure the speed of light like the rotating mirror, short pulse and periodic modulation methods.

The principle of the periodic modulation method, now used in the UPB student labs to measure the velocity of light in air, is to periodically modulate, in intensity, a light wave,

with a piezoelectric modulator, stabilized in frequency (f. e. $\nu = 60.000$ MHz), and to let the modulated signal to propagate through the studied transparent material (air), supposed to be homogenous.

In his way, the phase difference introduced by propagation between the light source and the detector is reduced $\sim 10^7$ times, but, nevertheless, this new difference of phase is too large to be measured with a simple oscilloscope.

The solution is to multiply this signal with another signal of approximately equal intensity, but with a frequency ν' , which is differing from the modulating signal ν with only ~ 100 kHz (precisely automatically measured). The resulting mixed signal may be monitored with a cheap oscillator, after filtering the higher frequency component, with the frequency equal to $\nu + \nu'$.

The author has managed to use the existent setup designed to measure the speed of light in air to offer sophomores and eventually, seniors, a laboratory work by which they may determine the difference of the refractive indices of the core and cladding of the optical fibers assembled in optical and light cables.

2. Theoretical approach

Let's consider a periodic light signal with intensity, I , periodically modulated:

$$I = I_0 + \Delta I_0 \cos(2\pi\nu t) \quad (1)$$

which may be used to measure the light speed.

The light signal is detected and measured using a receiver (transducer) which converts it into a variable electrical tension, U , proportional to the received intensity, which is described by the equation:

$$U = U_0 \cos(2\pi\nu t) \quad (2)$$

If the distance between the transmitter and the receiver is changed with the distance d (Fig. 1), then a t delay would appear, equal to:

$$\Delta t = \frac{d}{c} \quad (3)$$

(where c is the speed of light in the propagation medium) and also a phase difference, $\Delta\phi$, equal to:

$$\Delta\phi = 2\pi\nu \cdot \Delta t = 2\pi \frac{\Delta t}{T} \quad (4)$$

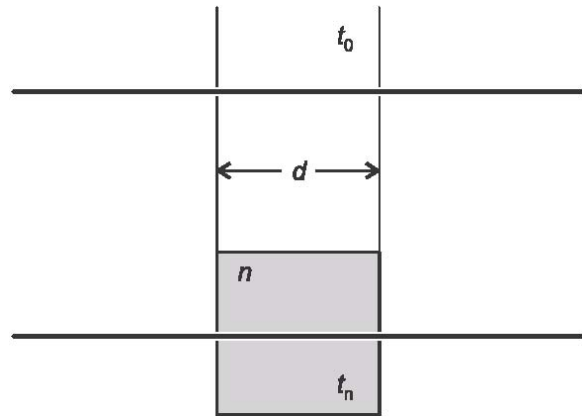


Figure 1. Propagation in vacuum (up) and in a material (down).

where ν is the modulation frequency of the light signal and T is the period of the modulation in intensity of the light signal.

This change may be measured as a phase shift $\Delta\phi$ between a reference and a receiver signal.

Neglecting some losses in intensity, the receiver measures a signal with a $\Delta\phi$ delay:

$$U = U_0 \cos(2\pi\nu t - \Delta\phi) \quad (5)$$

Using (3) and (4), the light speed, c , may be determined in the material of thickness d , measuring the phase difference $\Delta\phi$:

$$c = \frac{d}{\Delta t} = \frac{d}{\Delta\phi} 2\pi\nu \quad (6)$$

If the modulation frequency, ν , is very high, then, even on short Δx distances, considerable $\Delta\phi$ phase differences may result.

In this experiment, where the modulation frequency, ν , is 60.000 MHz [1], for a distance d of 5 m results a phase difference of:

$$\Delta\phi = 2\pi \cdot 60 \cdot 10^6 \text{ s} \cdot \frac{5\text{ m}}{3 \cdot 10^8 \text{ m/s}} = 2\pi \text{ rad} \quad .$$

But, this high frequency could not be displayed on a low frequency oscilloscope.

Because this experiment uses a simple oscilloscope to determine the phase difference $\Delta\phi$, the solution has been to electronically multiply the signal received by the receiver, with an other signal with the frequency of $\nu' = \sim 59.900$ MHz, displayed in real time by the equipment, so that:

$$U = U_0 \cos(2\pi\nu \cdot t - \Delta\phi) \cdot \cos(2\pi\nu' t) \quad (7)$$

Using the trigonometric relationship:

$$\cos a \cdot \cos b = \frac{1}{2}[\cos(a + b) + \cos(a - b)], \quad (8)$$

the resulting signal becomes the summation of two signals, one with the frequency of $(\nu + \nu')$ and the other one with the frequency ν_1 ($\nu_1 = \nu - \nu'$).

By using a filter to cut high frequencies, the high frequency component in the summation may be suppressed, so the signal displayed will be:

$$U = \frac{1}{2}U_0 \cos(2\pi\nu_1 t - \Delta\varphi). \quad (9)$$

This signal may be displayed on the simple oscilloscope used, because his frequency is only ~ 100 kHz. The multiplication of the two signals does not alter the phase difference $\Delta\varphi$.

After multiplication, this $\Delta\varphi$ may be associated to a measurable apparent time difference Δt_1 . The period of this last signal, T_1 , can be read on the display and the phase difference can be computed using the relationships:

$$\Delta\varphi = 2\pi \frac{\Delta t_1}{T_1} = 2\pi \frac{\Delta t}{T}, \quad (10)$$

where Δt_1 is the apparent change in the propagation time.

The actual time difference, Δt , introduced by the propagation of light along the distance d , in vacuum, may be obtained from,

$$\frac{\Delta t}{T} = \frac{\Delta t_1}{T_1}, \text{ as:}$$

$$\Delta t = \Delta t_1 \frac{T}{T_1} = \frac{\Delta t_1}{\nu T_1}. \quad (11)$$

By using equations (6) and (11), the final equation for the light speed in the material may be obtained:

$$c = \frac{d}{\Delta t} = \frac{d}{\Delta t_1} \cdot \frac{T_1}{T} = \frac{d}{\Delta t_1} \cdot T_1 \cdot \nu. \quad (12)$$

In order to determine more exactly the phase difference there is used a reference signal.

This reference signal is synchronized electronically, with the signal from the transmitter, and also it is multiplied with the same $\nu' = 59.900$ MHz signal and filtered exactly like the one from the receiver. Because the delay introduced by the wires and the experimental equipment are unknown but can't be neglected, firstly the transmitter is placed at the distance s from the receiver.

The reference signal is synchronized with the signal from the receiver, by adjusting a compensation phase difference system to have the two signals, in phase. After this synchronization, the transmitter is placed at the distance $\Delta s = s + d$ from the receiver. The phase difference that can be seen on the display (Fig. 2) is caused by the delay time Δt , only.

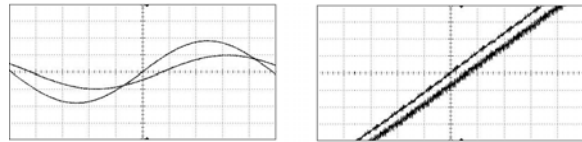


Figure 2. Reference and receiver signals for propagation in a material (left) and a detail for a thin cylinder or a light cable (right).

3. Measurement of a speed of light in a homogenous transparent material

The velocity of light c_n in a medium depends on the refractive index n of that medium. This material-dependent quantity relates the velocity of light in the medium c_n , to that in vacuum, $c_0 = 2.998 \cdot 10^8$ m/s, via:

$$c_n = \frac{c_0}{n}. \quad (13)$$

Different velocities of light correspond to different propagation times, along the same geometrical path length.

The propagation time in a medium of refractive index n is (see 6):

$$t_n = \frac{d}{c_n} \quad (14)$$

whereas the propagation time in vacuum, t_0 , is (see equation 3)

$$t_0 = \frac{d}{c_0} \quad (15)$$

By introducing the definition

$$\Delta't = t_n - t_0 \quad (16)$$

and using the equations (14), (15) and (16), the velocity of light in the medium, c_n , may be computed as being:

$$c_n = \frac{c_0}{n} = \frac{c_0}{1 + \Delta't * c_0 / d} \quad (17)$$

From Eqs. (11) and (17) one, finally, finds:

$$c_n = \frac{c_0}{n} = \frac{c_0}{1 + \Delta't_1 * c_0 / (d * v * T_1)} \quad (18)$$

and the refractive index (equations 13 and 17) is

$$n = 1 + \frac{\Delta't * c_0}{d} = 1 + \Delta't_1 * c_0 / (d * v * T_1) \quad (19)$$

The quantities d and $\Delta't_1$ are to be measured in the laboratory after the alignment and the optimization of the adjustment and of synchronization of the devices [1, 2].

The period T_1 is displayed by the equipment.

The frequency of intensity modulation, v , is precisely known (60.000 MHz).

The results obtained have a convenient precision if the transmitter and the receiver are stable mechanically, thermally and optically. It is to start the experiment after both transmitter and receiver have been turned on and the components of the setup are fastened since half an hour.

For the purpose of this laboratory work and considered the actual level of relative errors, the propagation in air may be considered to a good approximation of the propagation in vacuum the refractive index of air differing from the refractive index of the vacuum with only a part in three thousand parts.

4. Measurement of the velocities of light in optical and light cables

The optical cables, largely used in communications, are bundles of optical fibers

assembled together. Single fibres are also used in communications and as remote sensors, to connect a non-fiberoptic sensor to a measurement system and for many other purposes.

An optical fiber is a cylindrical dielectric waveguide that transmits light along its axis, by the process of total internal reflection (Fig. 3).

The fiber consists of a core surrounded by a cladding layer. To confine the optical signal in the core, the refractive index of the core must be larger than that of the cladding. The boundary between the core and cladding may either be abrupt, in step-index fiber (like in these experiments done in UPB General Physics Laboratory) or gradual, in graded-index fiber.

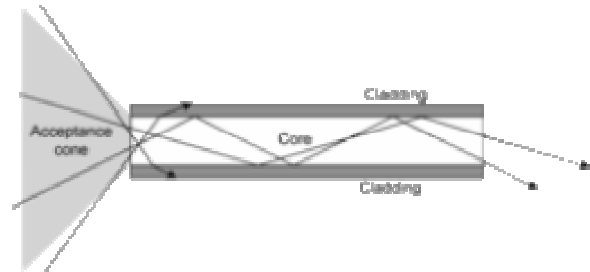


Figure 3. Propagation of light through a multi-mode optical fiber.

When light travelling in an optically dense medium hits a boundary at a steep angle, measured from the normal, larger than the "critical angle", θ_c , for the boundary (see "acceptance cone" in Fig. 3),

$$\theta_c = \arcsin n_{cl}/n_{co} \quad , \quad (20)$$

the light will be completely reflected.

Light travels along the fiber bouncing back and forth off of the boundary. This range of angles which ensure total reflection is called the acceptance cone of the fiber. The size of this acceptance cone is a function of the refractive index difference between the fiber's core and cladding. The sine of this maximum angle, θ_c , is the numerical aperture (NA) of the fiber. Fiber with a larger NA requires less precision to splice and work with than fiber with a smaller NA.

Fiber with large (greater than $10 \mu\text{m}$) core diameter may be analyzed by geometric optics.

A high numerical aperture allows light to propagate down the fiber in rays both close to the axis and at various angles, allowing efficient coupling of light into the fiber. However, this high numerical aperture increases the amount of

dispersion as rays at different angles have different path lengths and therefore take different times to traverse the fiber. A low numerical aperture may therefore be desirable for the determination of light speeds in core and in cladding in a students' laboratory.

As an optical waveguide, modelled using the wave equation, the fiber supports one or more confined transverse modes by which light can propagate along the fiber. Fiber supporting only one mode is called single-mode fiber (less than 10 micrometers in diameter). The behaviour of larger-core multimode fiber can also be modelled using the wave equation, which shows that such fiber supports more than one mode of propagation (hence the name). The results of such modelling of multi-mode fiber approximately agree with the predictions of geometric optics, if the fiber core is large enough to support more than a few modes. Optical cables made out of such fibers are chosen for the laboratory work.

The waveguide analysis shows that the light energy in the fiber is not completely confined in the core. Instead, especially in single-mode fibers, a significant fraction of the energy in the bound mode travels in the cladding as an evanescent wave, which may impair the clarity of the curve on the display corresponding to the propagation in the cladding. Therefore, thorough choice of the type of the optical fibers used for measuring the velocities is compulsory.

The properties of the fibers are tailored to a wide variety of applications. Glass optical fibers are almost always made from silica. Like other glasses, these glasses have a refractive index of about 1.5. Typically the difference between core and cladding is 1-2 one percent.

In practical fibers, the cladding, also transparent, is usually coated with a tough resin *buffer* layer, which may be further surrounded by a *jacket* layer, usually plastic, not transparent.

These layers add strength to the fiber but do not contribute to its optical wave guide properties.

Optical fiber can be used as a medium for telecommunication and networking because it is flexible and can be bundled as cables.

Because the aligning of an optical cable or its placing in contact with the device to which it is to couple is much easier than the aligning of fiber, in the newly developed laboratory work are used optical cables with a diameter comparable

with the diameter of the diameter of the laser pencil used.

Although fibers can be made out of transparent plastic, glass, or a combination of the two, the fibers used in long-distance telecommunications applications are always made out of glass, because of the lower optical attenuation.

For the new lab work there have been used optical fibers made out of glass.

5. The determination of refractive indices and velocities of light in core and cladding of optical fibers forming optical cables.

The optical cable, with the entry and exit area polished, is fastened on a bank, connecting source and detector. Its length is measured (d).

The news in the standard procedure is the appearance of double sinusoid as the signal received by the propagation of the modulated light, due to different refractive indices and speeds of light in core and respectively, in cladding of the optical fibers of the cable.

There are measured **two** new delays, corresponding to propagation through core and cladding:

$$\Delta t_{co} = t_{co} - t_0 \quad (21)$$

and

$$\Delta t_{cl} = t_{cl} - t_0 \quad (22)$$

If we define Δn , the difference of the refractive indices of core and cladding, as:

$$\Delta n = n_{co} - n_{cl} \quad (23)$$

and make use of the equations (19) and (23), we find the difference, Δn , of refractive indices equal to:

$$\Delta n = (\Delta t_{cl} - \Delta t_{co}) * c_0 / (d * v * T_1) \quad (24)$$

The two velocities of light are (equations 17, 21 and 22):

$$c_{co} = \frac{c_0}{n_{co}} = \frac{c_0}{1 + \Delta t_{co} * c_0 / (d * v * T_1)} \quad (25)$$

$$c_{cl} = \frac{c_0}{n_{cl}} = \frac{c_0}{1 + \Delta t_{cl} * c_0 / (d * v * T_1)} \quad (26)$$

Attention must be paid that the cable be well aligned and fastened to have a correctly measured d , considered in the computing formulae.

6. Processing of the experimental data

The results are computed using the experimentally found data, known quantities and the relationships (24), (25) and (26).

The absolute and the relative standard errors are found by using error compounding formulae.

7. Results

The results obtained by the students have been:

$$\Delta n = 0.20 \pm 0.02$$

$$n_{co} = 1.46 \pm 0.02$$

$$n_{cl} = 1.44 \pm 0.02$$

$$c_{co} = (2.05 \pm 0.03) \cdot 10^8 \text{ m/s}$$

$$c_{co} = (2.08 \pm 0.03) \cdot 10^8 \text{ m/s}$$

8. Conclusions

One important resource to cheaply improve Experimental Physics training, in schools and universities and at the same time, to offer more experimental opportunities to the interested

pupils and students, is to use the existing standard teaching laboratory equipment to perform experiments outside the initial scope the equipment has been designed, produced, advertised and sold for.

These new experiments may refer to: larger ranges, a better precision, smaller quantities, more complex investigated objects, new phenomena (eventually interdisciplinary) and higher levels of the processing of experimentally got data.

For example, the new application of the standard equipment, designed for the measurement, in a General Physics teaching laboratory, of the velocity of light in air, has been used to study the two velocities of propagation of light in a light cable and to determine the difference of the refractive indices of core and cladding of the optical fibers, assembled in a light cable, without any new financial or material investment, based only upon creative investment, a resource at disposal in every laboratory, *rich or poor*.

10. References

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